

**SUBJECT: Mathematics**

Part-I

Q1 Answer the following questions:

(2 x 10)

- a) Find the Laplace transform of  $( )^3 \cosh 2t$ .
- b) State first shifting theorem.
- c) Give an example of a periodic function, which have no fundamental period.
- d) Define even and odd functions.
- e) Define random variables.
- f) Check whether  $f(x) = \begin{cases} 1/5 & 0 \leq x < 5 \\ 0 & \text{elsewhere} \end{cases}$  is a probability density function?
- g) Given that X has a Poisson distribution with variance 0.5, calculate  $P(X = 3)$ .
- h) Define Binomial distribution.
- i) Define confidence interval.
- j) Define correlation coefficient. Write the range of it.

**Part-II**

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) (6 x 8)

- a) Find the Inverse Laplace transform of  $(s^2 + 1)^{-1}$ .
- b) State and prove Second shifting theorem.
- c) Solve the integral equation  $\int_0^1 \sinh t y(t) dt = 1 - y(1)$ .
- d) Using separation of variables solve the partial differential equation  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ ;  $z(0, y) = 0$ ,  $z(1, y) = 0$ .
- e) Find the Fourier series of the function  $f(x) = \begin{cases} -x & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$ .
- f) Find the Fourier coefficients of the function  $f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$  and  $f(x) = 2$ .
- g) If  $f(x) = \begin{cases} 3 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$  is the p.d.f. of X, then compute  $E(X)$ .

h) Find the mean of the random variable X whose probability density is given by  $f(x) = \begin{cases} 11 - 2.4x & 0 \leq x \leq 8 \\ 0 & \text{elsewhere} \end{cases}$

i) Find a formula for the probability distribution of the total number of heads obtained in four tosses of a balanced coin.

j) Show that mean and variance of Poisson distribution are same.

k) Apply maximum likelihood estimate for the parameter  $\mu$  of the normal distribution with known variance  $\sigma^2$ .

l) Write a short note on random sampling.

### Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 a) Using Laplace transform Solve the initial value problem  $y'' + y' - 1 + y = 2, y(0) = 1, y'(0) = 1$ .

(8x2)

b) Using convolution determine the inverse Laplace transform of  $(s^2 + 1)^{-2}$ .

Q4 a) Determine the solution of one dimensional heat equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  subject to boundary conditions  $u(0, t) = u(l, t) = 0, u(x, 0) = f(x) > 0$  and the initial conditions  $u(x, 0) = f(x)$  (l being the length of the bar).

(8x2)

b) A string is stretched and fastened to two points 'l' apart. Motion is started by displaying the string into the form  $y(x, 0) = \sin\left(\frac{\pi x}{l}\right)$  from which it is released at time  $t = 0$ . Find the displacement of any point on the string at a distance of  $x$  from one end at time  $t$ .

Q5 a) Write short notes on

(i) Probability distribution functions.

(ii) Cumulative probability distribution functions.

(8x2)

b) Compute the mean, variance, and standard deviation of Normal distributions

Q6 a) Find the Maximum likelihood estimation for Binomial population.

(8x2)

b) Find the regression line on  $y$  for the data  $(-2, 3.5), (0, 1.5), (2, 1), (4, -0.5), (6, -1)$ .