

Subject: Mathematics

Part-I

- a) Write the definition of a basis for a vector space V .
- b) Under what condition a nonhomogeneous system of m linear equations in n unknowns will have no solution?
(2 x 10) c) (1 0 0 If $A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$, what are the eigenvalues of the matrix A ?
- d) Eigen values of skew-symmetric matrices are either 0 or $\pm i$. Express the straight line parametrically which passes through the point $(2, -1, 4)$ in the direction of the vector $(1, 2, -1)$. f Find curl of $F = x^2i + y^2j + z^2k$.
- g) State Green's Theorem in a plane.
- h) Find the surface normal N to the surface $f(x, y, z) = x^2 + y^2 - z^2$.
- i) What is the fundamental period of the function $\cos x$?
- J) Define Fourier series of a function $f(x)$ in $(-\pi, \pi)$.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of
(6 x 8)

Twelve)

- a) Solve the equations $4y + 3z = 8$; $4x - 2z = 10$; $3x + 2y = 5$ by any suitable method. 42 emeiete r ano 3 3 2
5 20 5
- c) Show that the product of two orthogonal matrix is orthogonal.
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- d) Diagonalize the matrix
- e) Show that the eigenvalues of Hermitian matrix is always real.
- f) Find the directional derivative of the function $f(x, y, z) = e^x + e^y + e^z$ at the point
 $P(-4, 2, 3)$ in the direction $a = [1, 2, 1]$
- g) Find the area bounded by the line $y = x$ and the curve $y = x^2$.
- h) Evaluate $\int_C F \cdot dr$ where $F = [1, y, z]$ and $C: r = [1, \cos t, \sin t]$ from $(0, 1, 0)$ to $C(\pi/2, 0, 1)$.

i) Use Stokes' theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x^2\mathbf{i} + 2x\mathbf{j} + 2z\mathbf{k}$ and S is $x^2 + y^2 = 1, z \geq 0$.

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the surface given by $z = k, -\pi < x < \pi$ Find the Fourier series of the given function $f(x) = -k, 0 < x < \pi$ Find the Fourier cosine transform and Fourier sine transform of $f(x) = 5, -1 < x < 1$ otherwise Show by Fourier integral that $\int_0^\infty \cos wx + w \sin wx \, dw = 0, x < 0$ $\int_0^\infty \frac{dw}{1+w^2} = \frac{\pi}{2}, x > 0$

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Solve the system of equations $2x_1 + x_2 - 2x_3 + 2x_4 = 5; 4x_1 + 5x_2 - 3x_3 + 6x_4 = 9; -2x_1 + 5x_2 - 2x_3 + 6x_4 = 4;$
 $4x_1 + 11x_2 - 4x_3 + 8x_4 = 2.$

(-2 -4 2

(16)

Q4 Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} -2 & 1 & 2 \\ 1 & -4 & 2 \\ 2 & 2 & 0 \end{bmatrix}$,

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Q5 Find the Fourier series of the function $S(x)$ with period 4. $[2x, 0 < x < 2$

Q6 Evaluate $\int_V \mathbf{F} \cdot d\mathbf{A}$ where $\mathbf{F} = [6x, 0, -2z]$, over the sphere $S: x^2 + y^2 + z^2 = 4$

(i) directly and (ii) using Gauss Divergence theorem.

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