

Part-I

Q1 Answer the following questions: (2 x 10)

- Find the length of the curve $y = \cos x$; $0 \leq x \leq \pi$.
- Determine the value of $\int_0^1 x^2 dx$.
- In the Taylor's series expansion of e^x about $x = 2$, find the coefficient of $x - 2$.
- A function $f(x) = x^3 - x + 1$ is defined in the closed interval $[-1, 1]$. Find the value of x in the open interval $(-1, 1)$ for which the mean value theorem is satisfied.
- Find the first order partial derivatives of $f(x, y) = x^2 + y^2 + \log x$.
- Define sufficient condition for relative maxima for a function of two variables.
- Check whether the set $S = \{(1,1,0), (0,1,0), (0,0,1)\}$ is LI or LD?
- Are the vectors $\vec{a} = (1,0, -1)$, $\vec{b} = (1,2,1)$, $\vec{c} = (0, -3,2)$ forms a basis for R^3 ?
- Let A and B be two square matrices of the same order and A is symmetric. Show that $AB + BA$ is also symmetric.
- State Caley-Hamilton theorem.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) (6 x 8)

- Find the entire length of the curve $x = a \cos t$, $y = a \sin t$; $0 \leq t \leq \pi$.
- Define improper integral. Test the convergency of $\int_1^\infty \frac{1}{x^2} dx$.
- Find the volume of a sphere of radius r .
- Discuss the applicability of Rolle's theorem to the function $f(x) = x^3 - 3x^2 + 2x$ in the interval $(-1,1)$ using mean value theorem.
- Expand $\tan^{-1} x$ in powers of x by Maclaurin's theorem.
- If $\int_0^1 x \sin x dx = a$, prove that $\int_0^1 x^2 \sin x dx = b$.
- Define maxima, minima, and saddle point. Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x^2 - 3y^2$.

i) Write the method of Lagrange's multipliers method. Find the extreme values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to $2x + 5y + z = 1$.

j) If $V = \{x, y, z\}$, then show that V is a vector space and find its basis and dimension.

k) Show that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix.

l) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 1 & 6 \\ 2 & 1 & 2 \end{pmatrix}$.

Part-III

Only Type Long Answer Questions (Answer Any Two out of Four)

Q3 a) Find the surface of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} + z^{2/3} = a$ about the x -axis. (8x2)

b) Find the Maclaurin's series for $f(x) = e^x$. Also find the Maclaurin's series of $g(x) = \cosh x$.

Q4 a) If $u = f(x, y, z)$, where $x^2 + y^2 + z^2 = r^2$, prove that $(\nabla^2 u) = 2u$. (8x2)

b) If $x = r \cos \theta \cos \phi$, $y = r \sin \theta \cos \phi$, $z = r \sin \theta \sin \phi$, show that $(\nabla^2 u) = \frac{1}{r^2} \frac{\partial}{\partial \theta} (r^2 \frac{\partial u}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} (\sin^2 \theta \frac{\partial u}{\partial \phi}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial r} (r^2 \frac{\partial u}{\partial r}) = \frac{\partial^2 u}{\partial r^2}$.

Q5 a) Solve the system of linear equation by Gaussian elimination method:

$$2x + y = 0, x + 2y + z = 0, x + 2y + 2z = 0, x + 2z = 5. \quad (8x2)$$

b) Let $U = \{(x, y, z) : x - 2y + z = 0\}$ and $V = \{(x, y, z) : x = y, z = 2x\}$ be subspaces of \mathbb{R}^3 . Then find basis and dimension of $U \cap V$.

Q6 a) Find inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 4 & 0 & 1 & 1 \end{pmatrix}$ (using Gauss Jordan elimination. (8x2)

b) Define orthogonal matrix. Discuss about the eigenvalue of it.